of. 10p

Barem clasa V-a

Sub I:

a) $u(S) = u(P_1) + u(P_2) + \ldots + u(P_{2011}) = 7$ (10p)

b) De la al doilea termen al sumei fiecare termen este multiplu de 6 $\Rightarrow S = 6K+1$ (10p)

c)
$P_1 = 1$ $\quad 1$
$P_2 = 2\cdot 3$ $\quad 1+2=3$
$P_3 = 4\cdot 5\cdot 6$ $\quad 1+2+3=6$
$P_4 = 7\cdot 8\cdot 9\cdot 10$ $\quad 1+2+3+4=10$

Pentru a obține cel mai mare factor din P_{100} calculăm
$1+2+3+\ldots+100 = 5050$ (5p)

Pentru a afla cel mai mic factor din P_{100} calculăm
$(1+2+3+\ldots+99)+1 = 4951$ (5p)

Sub II:

1. $x = 10^{2012} - 2012 = \underbrace{99\ldots99}_{2008 \text{ ori}}7988$ (5p)

a) $u(x) = 8 \Rightarrow$ nu este p.p. (5p)

b) suma cifrelor $= 2009\cdot 9 + 2\cdot 8 + 7 = 18104$ (10p)

2. $A = 9^{2011} = (1+8)\cdot 9^{2010} = 9^{2010} + 8\cdot 9^{2010} = (9^{670})^3 + (2\cdot 9^{670})^3$ (10p)

$B = 10^{2011} = 10\cdot 10^{2010} = 10^{2010} + 9\cdot 10^{2010} = (10^{1005})^2 + (3\cdot 10^{1005})^2$ (10p)

Sub III:

	A	B
	x	y
I pas	$x-2y$	$2y$
al II-lea pas	$2x-2y$	$3y-x$
al III-lea pas	$3x-5y$	$6y-2x$

(10p)

$3x - 5y = 48$
$6y - 2x = 48$

$x + y = 96$
$2x + 2y = 192$
$6y - 2x = 48$

$8y = 240$
$y = 30$
$x = 66$

(10p)

$\boxed{of: 10p}$

Barem clasa $\overline{VI}$-a

Sub I: $\frac{a}{\frac{1}{20}} = \frac{b}{\frac{1}{10}} = \frac{c}{\frac{1}{12}} = \frac{S}{\frac{14}{60}} \Rightarrow a = \frac{3S}{14}$; $b = \frac{3S}{7}$; $c = \frac{5S}{14}$ (10p)

$\frac{x}{\frac{1}{10}} = \frac{y}{\frac{1}{5}} = \frac{z}{\frac{1}{3}} = \frac{S}{\frac{19}{30}} \Rightarrow x = \frac{3S}{19}$; $y = \frac{6S}{19}$; $z = \frac{10 \cdot S}{19}$ (10p)

Obs. că: $\left.\begin{array}{l} a > x \\ b > y \\ c < z \end{array}\right| \Rightarrow$ $z - c = 90 \Rightarrow S = 532$ (10p)

$a = 114$; $b = 228$; $c = 190$

Sub II:

① $A = 1005 \cdot 1006$

$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \ldots + \frac{1}{2009\cdot 2011} = \frac{1005}{2011}$ (10p)

$A + x = 1005 \cdot 1006 + x \Rightarrow x = 1006$ (10p)

② $x_1 = 1$, $x_2 = 3$, $x_3 = 5$, $x_4 = 7$, $\ldots$, $x_n = 2n-1$

$\Rightarrow 1 + 3 + 5 + \ldots + 2n - 1 = n^2$ p.p. (10p)

Sub III: desen corect (5p)

$3(a + 2c) + b = 51 \Rightarrow b = 3$, $a = 2$, $c = 7$ (5p)

a) $m\widehat{COA} = 110^\circ$ (10p).

b) $m\widehat{DOE} = 62^\circ 30'$

$m\widehat{COD} = 41^\circ 40'$

$m\widehat{EOA} = 145^\circ 50'$

$\Rightarrow$ măsura unghiului căutat este $156^\circ 15'$ (10p)

$\boxed{\text{of}: 10p}$

Barem clasa VII-a

Sub I:

a) $a = \frac{2}{3}$; (5p)

$b = 36$ (10p)

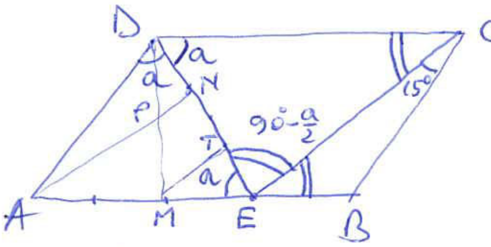
b) $A = \{1, 6\}$ (10p)

$3a \notin A$; $\sqrt{b} \in A$ (5p)

Sub II:

1:
$$A = \frac{3}{2} + \frac{9}{4} + \frac{15}{6} + \cdots + \frac{3453}{1152} + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{576}\right)$$
$$= \frac{3}{2}\left(1 + \frac{3}{2} + \frac{5}{3} + \cdots + \frac{1151}{576}\right) + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{576}\right)$$
$$= \frac{3}{2}(\underbrace{2 + 2 + \cdots + 2}_{576 \text{ ori}}) = 3 \cdot 576 = (3 \cdot 4)^3 = 12^3$$ (20p)

2 desen corect (5p)



a) Fie $m\widehat{ADC} = 2a \Rightarrow$

$\Rightarrow m\widehat{ADE} = m\widehat{EDC} = m\widehat{AED} = a$ (alt. int.)

$\Rightarrow m\widehat{DEC} = m\widehat{BEC} = 90° - \frac{a}{2} \Rightarrow m\widehat{DCE} = 90° - \frac{a}{2}$ (alt. int.)

$\Rightarrow 2a + 90° - \frac{a}{2} + 15° = 180° \Rightarrow a = 50° \Rightarrow m\hat{A} = m\hat{C} = 80°$, $m\hat{B} = m\hat{D} = 100°$ (10p)

b) fie $T \in (ED)$ a.î. $\frac{ET}{TD} = \frac{1}{3} \Rightarrow \frac{ET}{TN} = \frac{1}{2} = \frac{EM}{MA} \overset{R.T.Th}{\Rightarrow} MT \parallel AN \Rightarrow$

$\Rightarrow PN \parallel MT \overset{T.Th}{\Longrightarrow} \frac{DP}{PM} = \frac{DN}{NT} = \frac{1}{2}$ (5p)

Sub III:

desen corect (5p)

$MN \parallel BD$

O centrul de greutate $\Rightarrow$ P mijl lui MN $\Big| \Rightarrow$ $[PM] \equiv [PN]$, $[OP] \equiv [PC]$, $MP = \frac{OD}{2}$, $PN = \frac{OB}{2}$ $\Big\} \Rightarrow$

$\Rightarrow [OB] \equiv [OD]$

O centrul de greutate $\Rightarrow 2OP = OA \Rightarrow [OA] \equiv [OC]$ $\Big\} \Rightarrow$

$\Rightarrow ABCD$ paralelogram (15p)

$\boxed{\text{of}: 10p}$ Barem clasa $\overline{\text{VIII}}$ – a

Sub I: a) $\left.\begin{array}{l} S_1+S_2=\sqrt{2011}-1 \\ 44^2<2011<45^2 \end{array}\right| \Rightarrow S_1+S_2\in(43,44)$ (20p)

b) S_1, S_2 au 1005 termeni, iar fiecare termen din S_2 este mai mic decât termenul corespunzător din $S_1 \Rightarrow S_2<S_1$

$S_3=\frac{1}{2}(S_1+S_2) \Rightarrow S_3$ este media aritmetica a lui S_1 și $S_2 \Rightarrow S_2<S_3<S_1$ (10p)

Sub II: $S=(ax^3yz-x^2y^2z^2)+(bxy^3z-abx^2y^2)+(cxyz^3-acx^2z^2)+(abcxyz-bcy^2z^2)$

$=(ax-yz)(x^2yz-bxy^2-cxz^2+bcyz)=$ (20p)

$=(ax-yz)(by-xz)(cz-xy)$

Sub III: $a^2-2abc+b^2c^2+b^2-2abc+a^2c^2+c^2-2abc+a^2b^2=0$

$(a-bc)^2+(b-ac)^2+(c-ab)^2=0$ (10p)

$\Rightarrow a-bc=b-ac=c-ab=0$

$\left.\begin{array}{l} a=bc \\ b=ac \\ c=ab \end{array}\right| \Rightarrow \left.\begin{array}{l} abc=(abc)^2 \\ abc>0 \end{array}\right| \Rightarrow \left.\begin{array}{l} abc=1 \\ bc=a \end{array}\right| \Rightarrow \begin{array}{l} a=1 \\ b=1 \\ c=1 \end{array}$

$\Rightarrow$ paralelipipedul este cub $\Rightarrow d=l\sqrt{3}=\sqrt{3}\in\mathbb{R}-\mathbb{Q}$ (10p)

Sub IV: desen corect (5p)

$\left.\begin{array}{l} m(\widehat{VH,(VAB)})=m(\widehat{VH,(VBC)})=m(\widehat{VH,(VAC)}) \Rightarrow H \text{ este centrul cercului înscris în } \triangle ABC \\ H \text{ ortocentru} \end{array}\right| \Rightarrow \triangle ABC$ echilateral (5p)

$HD=h\,tg\,\alpha \Rightarrow AD=3h\,tg\,\alpha$

$AB=l=\frac{2AD}{\sqrt{3}}=2\sqrt{3}h\,tg\,\alpha$ (5p)

$VA^2=VH^2+AH^2 \Rightarrow VA^2=h^2(1+4tg^2\alpha)$

$VA\perp VB$ (de ex) $\Leftrightarrow VA^2+VB^2=AB^2$

$2h^2(1+4tg^2\alpha)=12h^2tg^2\alpha$ (5p)

$\Rightarrow tg\,\alpha=\frac{\sqrt{2}}{2}$