SOLUTII Clasa a VIII-a

1. a) $x^2 + y^2 + z^2 \geq xy + xz + yz \Leftrightarrow (x-y)^2+(x-z)^2 + (y-z)^2 \geq 0$ evident

b) $x^4 + y^4 + 1=(x^2)^2+(y^2)^2+1 \geq x^2y^2+x^2+y^2$

$x^2y^2+x^2+y^2 \geq xy \cdot x + xy \cdot y + xy = xy(x+y+1)$

2. a) n=0 si n=1 sunt solutii

Pentru $n \geq 2$, $\underbrace{2^n - 1 = 4k - 1}_{\vdots\, 4} = 4p+3 \neq$ patrat perfect

b) $\dfrac{a-b\sqrt{2011}}{b-c\sqrt{2011}} = x \in Q \Rightarrow$

$a-b\sqrt{2011} = xb - xc\sqrt{2011} \Rightarrow xc\sqrt{2011} - b\sqrt{2011} = xb - a \Rightarrow$

$\sqrt{2011}\,\underbrace{(xc-b)}_{\in Q} = \underbrace{xb-a}_{\in Q} \Rightarrow$

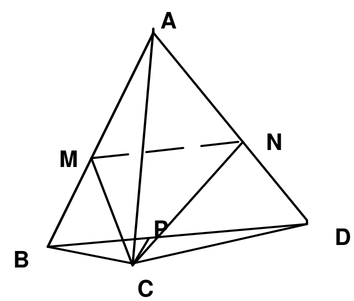
$xc-b = 0 \Rightarrow xc=b \Rightarrow x=\dfrac{b}{c}$

si

$xb-a = 0 \Rightarrow xb=a \Rightarrow x=\dfrac{a}{b}$

$\Rightarrow \dfrac{b}{c} = \dfrac{a}{b} \Rightarrow ac=b^2, b \in N \Rightarrow$ ac=p.perfect

3.



ΔABC : (CM bisectoare $\Rightarrow \dfrac{AM}{MB} = \dfrac{AC}{BC}$

ΔACD : (CN bisectoare $\Rightarrow \dfrac{AN}{ND} = \dfrac{AC}{CD}$

$(BC) \equiv (CD)$

$\Rightarrow \dfrac{AM}{MB} = \dfrac{AN}{ND} \Rightarrow$ Reciproca Th. Thales $\Rightarrow MN \parallel BD$

ΔBCD isoscel
(CP bisectoare $\Rightarrow$ (CP) inaltime $\Rightarrow CP \perp BD$

$MN \parallel BD$

$\Rightarrow CP \perp MN$

4. Fie N mijlocul (BC)

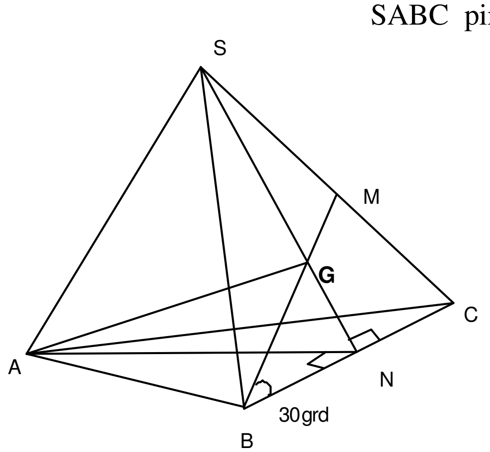
ΔSBC : (BM) si (SN) mediane
$BM \cap SN = \{G\}$ $\Rightarrow$ G= centrul de greutate $\Rightarrow SG = 2GN$

ΔBNG : m(N) = 90°
m (B) = 30° $\Rightarrow BG = 2GN$

$\Rightarrow SG=BG \Rightarrow SN=BM \Rightarrow$

$\Rightarrow \Delta SBC$ isoscel : $(CS) \equiv (CB)$
SABC piramida regulata $\Rightarrow$ SABC tetraedru regulat

d(A,SBC) = AG



ΔAGN : m(G)= 90° , $AG = \dfrac{a\sqrt{6}}{3}$