BAREM – cl. a V-a

oficiu 10p

Borodi S.

Sub I: $A = 1\cdot 2\cdot 3\cdot \ldots \cdot 2009\cdot 2010 - 29 + 29 - 15$ 10p

$= 29\cdot(1\cdot 2\cdot 3\cdot \ldots \cdot 28\cdot 30\cdot \ldots \cdot 2009\cdot 2010 - 1) + 14$, $14 < 29$ 10p

$\overset{\text{T.Împ.R}}{\Longrightarrow}$ Rest $= 14$. 10p

Sub II:

1. $13(1+3+5+\ldots+25) = 13\cdot\left(\frac{25+1}{2}\right)^2$ 15p

$= 13\cdot 13^2 = 13^3$ 5p

$\Rightarrow 13^3 = (\overline{ab}+1)^3 \Rightarrow \overline{ab} = 12$

2. Cele trei persoane au împreună acum: $99 - 3\cdot 4 = 87$ ani 5p

Vârsta tatălui: $(87-9):2 = 39$ ani 5p

Mama + fiul au împreună $39+9$ ani $= 48$ ani

Peste 4 ani vor avea $48+8 = 56$ ani

Fiul va avea o treime din vârsta mamei $56:4 = 14$ ani 5p

În prezent fiul are $14-4 = 10$ ani 5p

Vârsta mamei $48-10 = 38$ ani

Sub III a) $x = 2010^{ad}\cdot 2010^{bc}\cdot 2010^{ac}\cdot 2010^{bd}$ 5p

$= 2010^{c(a+b)}\cdot 2010^{d(a+b)}$ 5p

$= 2010^{(a+b)(c+d)} = 2010^{5\cdot 16} = 2010^{80}$ 5p

b) Fie numerele naturale consecutive:

$\alpha, \alpha+1, \alpha+2, \ldots, \alpha+2009$

$S = \alpha + (\alpha+1) + (\alpha+2) + \ldots + (\alpha+2009) = 2010\cdot\alpha + 2009\cdot 1005$ 2p

$\Rightarrow 2010^{80} - 1005 = 2010\alpha + 2009\cdot 1005$ 2p

$2010^{80} - 2010\cdot 1005 = 2010\cdot\alpha$ 1p

$\Rightarrow \alpha = 2010^{79} - 1005$

BAREM - cl. VI-a

oficiu — 10p

Sub I:

1) Fie $x, x+1, x+2, \ldots, x+2009$ numerele consecutive

$$\Rightarrow \frac{x}{6} = \frac{x+2009}{12} \Rightarrow x = 2009$$ — 10p

$S = 2009 + (2009+1) + (2009+2) + \ldots + (2009+2009)$

$= 2010 \cdot 2009 + (1+2+3+\ldots+2009)$ — 5p

$= 2009 \cdot (2010 + 1005) = 2009 \cdot 3015 = 6057135$ — 5p

2) $7 \cdot 1, 7 \cdot 2, 7 \cdot 3, \ldots, 7 \cdot 286$ - multipli de 7 — 5p

$13 \cdot 1, 13 \cdot 2, 13 \cdot 3, \ldots, 13 \cdot 154$ - multipli de 13 — 5p

$7 \cdot 13 = 91 \Rightarrow 91 \cdot 1, 91 \cdot 2, 91 \cdot 3, \ldots, 91 \cdot 22$ - multipli de 91 — 5p

$\Rightarrow 2008 - (286 + 154 - 22) = 2008 - 418 = 1690$ — 5p

Sub II:

Elementele din A sunt de forma $\frac{2010+x}{11+x}$, $x \in \{0,1,2,\ldots\}$ — 7p

$\frac{2010+x}{11+x} = 1 + \frac{1999}{11+x} \in \mathbb{N} \Rightarrow 1999 \,\vdots\, (11+x)$

Se demonstrează că 1999 este număr prim — 5p

$\Rightarrow 11+x = 1$, $x \notin \mathbb{N}$ sau $11+x = 1999$, $x = 1988 \Rightarrow A \cap \mathbb{N} = \{2\}$ — 3p

Sub III:

1. Fixăm un vârf A, fapt ce se poate realiza în 2010 moduri, al doilea vârf B, ce se poate face în $(2010-1)$ moduri iar al treilea vârf C se poate face în $(2010-2)$ moduri — 5p

Deci avem $2010 \cdot 2009 \cdot 2008$ triunghiuri fără a ține cont de ordinea literelor — 5p

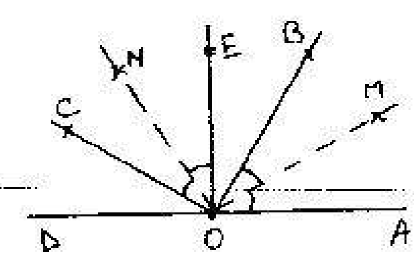
$\Rightarrow (2010 \cdot 2009 \cdot 2008) : 6 = 1351414120$ triunghiuri — 5p

2. Desen corect — 5p

a) Unghiurile $\widehat{AOB}$ și $\widehat{COE}$ au același complement pe $\widehat{BOE}$ deci sunt congruente — 10p

b) $m\widehat{MON} = m\widehat{BOE} + \frac{m\widehat{AOB}}{2} + \frac{m\widehat{COE}}{2}$ — 5p

$= 90^\circ$



BAREM – cl $\overline{VII}$ – a

oficiu — 10p

Sub I: 1. $\left.\begin{array}{l} a = 3k \\ b = 4k \\ c = 2k \end{array}\right| \Rightarrow$ Membrul stâng este $= \frac{1}{2} + \frac{4}{5} + \frac{2}{7} = \frac{111}{70} = 1{,}58\ldots$ — 10p

$30 \cdot \left(\frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \frac{1}{10 \cdot 13} \right) = 10 \left(\frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \frac{1}{10} - \frac{1}{13} \right) = 10 \cdot \frac{9}{52} = 1{,}73\ldots$ — 10p

$1{,}58 < 1{,}73$

2. Fie x – nr. de km parcurși (lungimea întregului drum) — 7p

$\Rightarrow \frac{x}{2} + \frac{1}{4} \cdot \frac{x}{2} + 9 = x \Rightarrow x = 24$ km. — 3p

I zi 12 km II zi 3 km

Sub II: 1. $\frac{3}{7} = 0{,}(428571) \Rightarrow$ a 2010-a zecimală este 1 — 10p

2. $\frac{n}{1 \cdot 2} + \frac{n-1}{2 \cdot 3} + \frac{n-2}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = n\left(\frac{1}{1} - \frac{1}{2}\right) + (n-1)\left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$ — 5p

$= n - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \ldots - \frac{1}{n+1} = \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{3}\right) + \ldots + \left(1 - \frac{1}{n+1}\right)$ — 5p

$= \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{n}{n+1} \Rightarrow x = 1$

Sub III: a) Desen corect — 5p

ΔBCE isoscel $\Rightarrow m\widehat{CBE} = 150^\circ \Rightarrow m\widehat{BCE} = m\widehat{BEC} = 15^\circ$ — 5p

$m\widehat{ACE} = m\widehat{ACB} - m\widehat{BCE} = 30^\circ \Rightarrow m\widehat{CAE} = 105^\circ \Rightarrow m\widehat{AEC} = 45^\circ$

$m\widehat{CAF} = m\widehat{BAF} - m\widehat{BAC} = 15^\circ \Rightarrow \Delta BCF$ isoscel $\Rightarrow m\widehat{CBF} = 30^\circ$

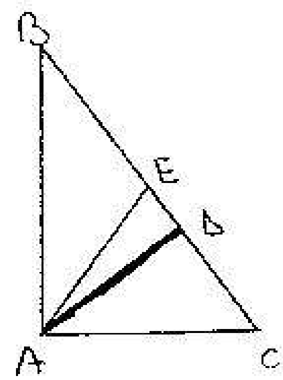
$\Rightarrow m\widehat{BCF} = 75^\circ \Rightarrow m\widehat{ACF} = m\widehat{BCF} - m\widehat{ACB} = 30^\circ$ — 5p

$\Rightarrow m\widehat{CFA} = m\widehat{BFC} + m\widehat{AFB} = 135^\circ$ — 5p

b) $m\widehat{ECA} = m\widehat{ACF} = 30^\circ \Rightarrow (CA$ bisect. $\sphericalangle ECF$ — 5p

Sub IV:

Desen corect — 5p



Fie $E \in (BC)$ a.î. $m\widehat{AEC} = 30^\circ \Rightarrow m\widehat{EAC} = 75^\circ$ — 5p

$\Rightarrow \Delta AEC$ isoscel $\Rightarrow EA = EC$

ΔAED dreptunghic $\Rightarrow AD = \frac{AE}{2} \Rightarrow AE = 8$ cm $= \frac{BC}{2}$

$EA = EC \Rightarrow EC = \frac{BC}{2} \Rightarrow BE = \frac{BC}{2}$ — 5p

$\Rightarrow AE = BE = CE$.

În ΔAEB avem $m\widehat{AEB} = 150^\circ$

$AE = BE \Rightarrow m\widehat{BAE} = m\widehat{ABE} = (180^\circ - 150^\circ) : 2 = 15^\circ$

$\Rightarrow m\widehat{BAC} = m\widehat{BAE} + m\widehat{EAC} = 15^\circ + 75^\circ = 90^\circ$ — 5p

$\Rightarrow \Delta ABC$ dreptunghic în A.

BAREM - cl $\overline{\text{VIII}}$ - a

oficiu 10 p

Sub $\underline{\text{I}}$:

1. $a = 2$

$b = 4$ 10 p

$m_a = 3$, $m_g = 2\sqrt{2}$, $m_h = \frac{8}{3}$ 5 p

2. fie $a = x^2 + y^2 \Rightarrow 2a = 2x^2 + 2y^2 = (x+y)^2 + (x-y)^2$ 10 p

$a^2 = (x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$ 5 p

Sub $\underline{\text{II}}$:

$(a+b)^2 \geq 4ab$, folosim ineg. mediilor $\Rightarrow \frac{1}{2}(a+b)^2 + \frac{1}{4}(a+b) =$

$= \frac{2(a+b)^2 + a + b}{4} \geq \frac{8ab + a + b}{4} = \frac{4ab + a}{4} + \frac{4ab + b}{4} \geq$

$\geq \frac{2\sqrt{4ab \cdot a}}{4} + \frac{2\sqrt{4ab \cdot b}}{4} = a\sqrt{b} + b\sqrt{a}$. 8 p

Avem egalitate dacă $a = b$, $4ab = a$ și $4ab = b \Rightarrow \left\langle \begin{array}{l} a=b=0 \\ a=b=\frac{1}{4} \end{array} \right.$ 2 p

Sub $\underline{\text{III}}$:

Obs. că $(1+\sqrt{2})^{100} = (3+2\sqrt{2})^{50} = (17+12\sqrt{2})^{25}$ 15 p

$\Rightarrow \frac{1}{3} \cdot 3 \cdot (17+12\sqrt{2})^{25} > (17 + 12 \cdot 1{,}4)^{25} = (33{,}8)^{25} > 32^{25} = 2^{125}$ 5 p

Sub $\underline{\text{IV}}$:

a) Desen corect 5 p

$\left.\begin{array}{l} A_{PBC} = 15 \text{ cm}^2 \\ A_{VBC} = 9\sqrt{5} \text{ cm}^2 \end{array}\right| \Rightarrow \frac{A_{PBC}}{A_{VBC}} = \frac{\sqrt{5}}{3}$ 10 p

$\sqrt{4} < \sqrt{5} < \sqrt{6} \quad | \cdot \frac{1}{3} \Rightarrow \frac{\sqrt{4}}{3} < \frac{\sqrt{5}}{3} < \frac{\sqrt{6}}{3}$ 5 p

b) $\left.\begin{array}{l} A_{VPM} = A_{VOM} - A_{POM} = 3 \text{ cm}^2 \\ A_{VPM} = \frac{VP \cdot PM \cdot \sin VPM}{2} \end{array}\right| \Rightarrow \sin VPM = \frac{3}{5}$ 10 p