BAREM DE CORECTARE SI NOTARE

Clasa a VII-a

1.

a) Tinand cont ca xy = $\frac{1}{z}$, yz = $\frac{1}{x}$, xz = $\frac{1}{y}$, inegalitatea devine

$x+\frac{1}{x}+y+\frac{1}{y}+z+\frac{1}{z}\geq 6$ care se obtine insumand inegalitatile cunoscute de tipul $x+\frac{1}{x}\geq 2$. (2p)

b) i) $(x+1)(y+1)=xy+x+y+1$

ii) Din xyz = 1 rezulta ca $\frac{1}{x}=yz$ deci

$\frac{1}{x}+y+z+1=yz+y+z+1=(y+1)(z+1)$

Analog $x+\frac{1}{y}+z+1=(x+1)(z+1)$ si $x+y+\frac{1}{z}+1=(x+1)(y+1)$

deci

$$\sqrt{(\frac{1}{x}+y+z+1)\left(x+\frac{1}{y}+z+1\right)\left(x+y+\frac{1}{z}+1\right)}=$$

$$\sqrt{(y+1)(z+1)(x+1)(z+1)(x+1)(y+1)}$$

$$=\sqrt{(x+1)^2(y+1)^2(z+1)^2}==(x+1)(y+1)(z+1)\in \boldsymbol{Q}.$$ (2p)

c) $\frac{\frac{1}{x}+y}{z}=\frac{\frac{1}{y}+z}{x}=\frac{\frac{1}{z}+x}{y}\Leftrightarrow\frac{yz+y}{z}=\frac{xz+z}{x}=\frac{xy+x}{y}$

$$\frac{yz+y}{z}=\frac{xz+z}{x}=\frac{xy+x}{y}=k$$

$\frac{yz+y}{z}=k\Rightarrow y=\frac{zk}{z+1}$ (1)

$\frac{xz+z}{x}=k\Rightarrow x=\frac{z}{k-z}$ (2)

$\frac{xy+x}{y}=k\Rightarrow xy+x=ky$ (3)

Din (1),(2),(3) rezulta

$\frac{z}{k-z}\cdot\frac{zk}{z+1}+\frac{z}{k-z}=k\frac{zk}{z+1}\Leftrightarrow z^2k+z^2+z=zk^2(k-z)\Leftrightarrow zk^2+zk+z=k^3-1\Leftrightarrow z(k^2+k+1)=(k-1)(k^2+k+1) de\ unde\ z=k-1$

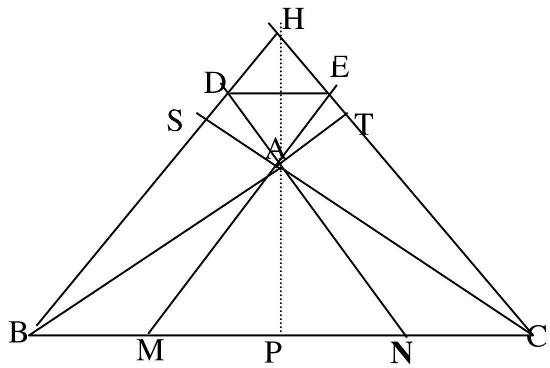
Inlocuind in (1),(2) obtinem y=k-1 si x=k-1

Tinem cont ca xyz=1 si obtinem $(k-1)^3=1$ deci k-1=1 adica k=2 si de aici x=y=z=1

2.a) prin calcul direct (2p)

b) $P^2=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2009}{2010}\cdot\frac{2009}{2010}<\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{2009}{2010}\cdot\frac{2010}{2011}=\frac{1}{2011}$, de unde rezulta concluzia. (5p)

3.



Soluţie:

a) În $\Delta HBC: BT\perp HC, CS\perp BH, BT\cap CS=\{A\}\Rightarrow A-ortocentrul\Delta HBC\Rightarrow HA\perp BC$.

b) $\Delta ABC-isoscel(AB=AC), AP\perp BC\Rightarrow[AP]-mediana, bisectoare\Rightarrow BP=PC,$
$\angle BAP\equiv\angle CAP.$ (2p)

$\Delta HBC: HP\perp BC, BP=PC\Rightarrow\Delta HBC-isoscel\Rightarrow\angle B\equiv\angle C;\angle BHP\equiv\angle CHP, dar$
$\angle ABC\equiv\angle ACB\Rightarrow\angle HBA\equiv\angle HCA.$
$m(\angle S)=m(\angle EAC)=90°(coresp.)\Rightarrow HB\parallel ME\Rightarrow DB\parallel ME\Rightarrow DH\parallel AE\Rightarrow$
$m(\angle T)=m(\angle BAD)=90°(coresp.)\Rightarrow HC\parallel DN\Rightarrow EC\parallel DN\Rightarrow HE\parallel AD\Rightarrow$
$AEHD-paralelogram(1)$

Dar, $\Delta BAD\equiv\Delta CAE(CU)\Rightarrow AD=AE(2), deci\ din\ (1) si\ (2)\Rightarrow AEHD-romb.$ (2p)

c) $DE\perp HA, BC\perp HA\Rightarrow DE\parallel BC\Rightarrow DE\parallel BM, dar DB\parallel ME(d.a.)\Rightarrow BDEM-paralelogram.$

d)
$ME\parallel BH(d.a.)\Rightarrow BHEM-trapez; daca BHEM-trapez\ isoscel.\Rightarrow\angle B\equiv\angle H,$
$dar\angle B\equiv\angle C(d.a.)\Rightarrow\angle B\equiv\angle C\equiv\angle H\Rightarrow\Delta HBC-echilateral\Rightarrow m(\angle B)=60°,$ (3p)
dar BA - $inaltime, deci, [BA-bisectoare\Rightarrow m(\angle ABC)=m(\angle ACB)=30°\Rightarrow m(\angle BAC)=120°.$

4.a)Din BM=MC si AB ∥ CF rezulta ca patrulaterul ABFC este paralelogram, deci AB=CF si AC ∥ BF.
Analog ABDE este paralelogram deci AB=DE si BD ∥ AE.
Cum ABCD e romb, $AC\perp BD$.
Din $AC\parallel BF$, $BD\parallel AE$ si $AC\perp BD$ rezulta $AE\perp BF$, deci $AP\perp BP$.

Din $AO \parallel BP$, $AP \parallel BO$ si $AP \perp BP$ rezulta APBO este dreptunghi deci AB=PO si AS=SB=PS=SO.
In triunghiul ABD, AS=SB si BO=OD deci SO e linie mijlocie, deci $SO \parallel AD$, rezulta $PQ \parallel AD \parallel BC$.
In triunghiul BCD, BO=OD si $OQ \parallel BC$ deci OQ e linie mijlociede unde deducem DQ=QC.
Din AB=DE=DC=CF si DQ=QC deducem ca EQ=QF si EF=3AB.
In triunghiul PEF, dreptunghic in P, PQ e mediana deci $PQ = \frac{1}{2}EF = \frac{3}{2}AB.$ (3p)
b) Paralelogramele ABFC si ABDE au aceeasi baza AB si inaltimi egale, deci au arii egale. $A_{ACFP} = A_{ABP} + A_{ABFC}$ *si* $A_{BDEP} = A_{ABP} + A_{ABDE}$ iar
$A_{ABFC} = A_{ABDE}$, *deci* $A_{ACFP} = A_{BDEP}$. (2p)
c) In triunghiul PEF, PQ e mediana iar PS=SO=OQ deci O este centru de greutatede unde rezulta ca triunghiurile EOF, EOP si FOP au arii egale. Astfel deducem ca $A_{EOF} = \frac{1}{3}A_{PEF}$ deci $A_{PEF} = 3A_{EOF}$. In triunghiul EOC, OD e mediana deci triunghiurile DOE si DOC au arii egale, analog triunghiurile FOC si DOC au arii egale de unde deducem ca $A_{EOF} = 3A_{DOC}$. Dar $A_{DOC} = \frac{1}{4}A_{ABCD}$. Rezulta ca $A_{PEF} = 3A_{EOF} = 9A_{DOC} = \frac{9}{4}A_{ABCD}$. De unde obtinem: $A_{ABCD} = \frac{4}{9}A_{PEF}$. (2p)