

Problema 1

$$8 + 8 \cdot 9 + 8 \cdot 9^2 + \dots + 8 \cdot 9^{2010} = 8 \underbrace{(1 + 9 + 9^2 + \dots + 9^{2010})}_S \dots\dots\dots 1p$$

$$\begin{cases} S = 1 + 9 + 9^2 + \dots + 9^{2010} \cdot 9 \\ 9S = 9 + 9^2 + \dots + 9^{2010} + 9^{2011} \end{cases}$$

$$8S = 9^{2011} - 1 \dots\dots\dots 1p$$

$$8S = x^{2011} - 1$$

$$9^{2011} - 1 = x^{2011} - 1 \Rightarrow x = 9 \dots\dots\dots 1p$$

$$A = 9 + 99 + 999 + \dots + \underbrace{999\dots9}_{2011 \text{ cifre}}$$

$$A = (10 - 1) + (100 - 1) + (1000 - 1) + \dots + (\underbrace{100000\dots0}_{2011 \text{ cifre de } 0} - 1) \dots\dots\dots 1p$$

$$A = 10 + 100 + \dots + \underbrace{100000\dots0}_{2011 \text{ cifre de } 0} - (1 + 1 + \dots + 1)_{2011}$$

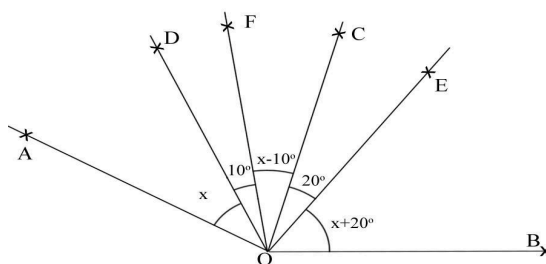
$$A = 11\dots10 - 2011 \dots\dots\dots 1p$$

$$A = 11\dots109099 \dots\dots\dots 1p$$

$$\text{Suma cifrelor} = 2007 + 27 = 2034 \dots\dots\dots 1p$$

$$\begin{array}{r} 111\dots111110 - \\ \underline{ 2011} \\ 111\dots109099 \\ \underline{ 2007} \end{array}$$

Problema 2



$$m(\angle AOB) = x \dots\dots\dots 1p$$

$$\left. \begin{aligned} m(\angle AOF) &= x + 10^\circ \\ m(\angle AOD) &= m(\angle DOC) \\ m(\angle FOC) &= x - 10^\circ \end{aligned} \right\} \dots\dots\dots 2p$$

$$m(\angle AOF) = m(\angle FOE) \Leftrightarrow x + 10^\circ = x - 10^\circ + m(\angle EOC) \Rightarrow$$

$$\Rightarrow m(\angle EOC) = 20^\circ \dots\dots\dots 1p$$

$$m(\angle BOE) = m(\angle EOD) \Leftrightarrow m(\angle BOE) = 10^\circ + x - 10^\circ + 20^\circ \dots\dots\dots 1p$$

$$\left. \begin{aligned} m(\angle AOD) + m(\angle DOF) + m(\angle FOC) &= m(\angle COE) + m(\angle BOE) \\ x + 10^\circ + x - 10^\circ &= 20^\circ + x + 20^\circ \\ 2x &= 40^\circ + x \\ x &= 40^\circ \end{aligned} \right\} \dots\dots\dots 1p$$

$$\text{Deci } m(\angle AOB) = (40^\circ + 20^\circ + 20^\circ) \cdot 2 = 160^\circ \dots\dots\dots 1p$$

Problema 3

$$\left. \begin{aligned} \frac{1}{2} &< \frac{2}{3} \\ \frac{3}{4} &< \frac{4}{5} \end{aligned} \right\} \dots\dots\dots 2p$$

$$\frac{99}{100} < \frac{100}{101}$$

$$P < P_1 \cdot P \Rightarrow P^2 < P \cdot P_1 \dots\dots\dots 1p$$

$$P \cdot P_1 = \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \dots \cdot \frac{99}{100} \cdot \frac{2}{3} \cdot \frac{4}{5} \cdot \dots \cdot \frac{98}{99} \cdot \frac{100}{101} = \frac{1}{101} \dots\dots\dots 1p$$

$$P \cdot P_1 = \frac{1}{101} < \frac{1}{100}$$

$$\left. \begin{aligned} P^2 &< P \cdot P_1 \\ P \cdot P_1 &< \frac{1}{100} \end{aligned} \right| \Rightarrow P^2 < \frac{1}{100} \dots\dots\dots 2p$$

$$\text{Deci } P < \frac{1}{10} \dots\dots\dots 1p$$